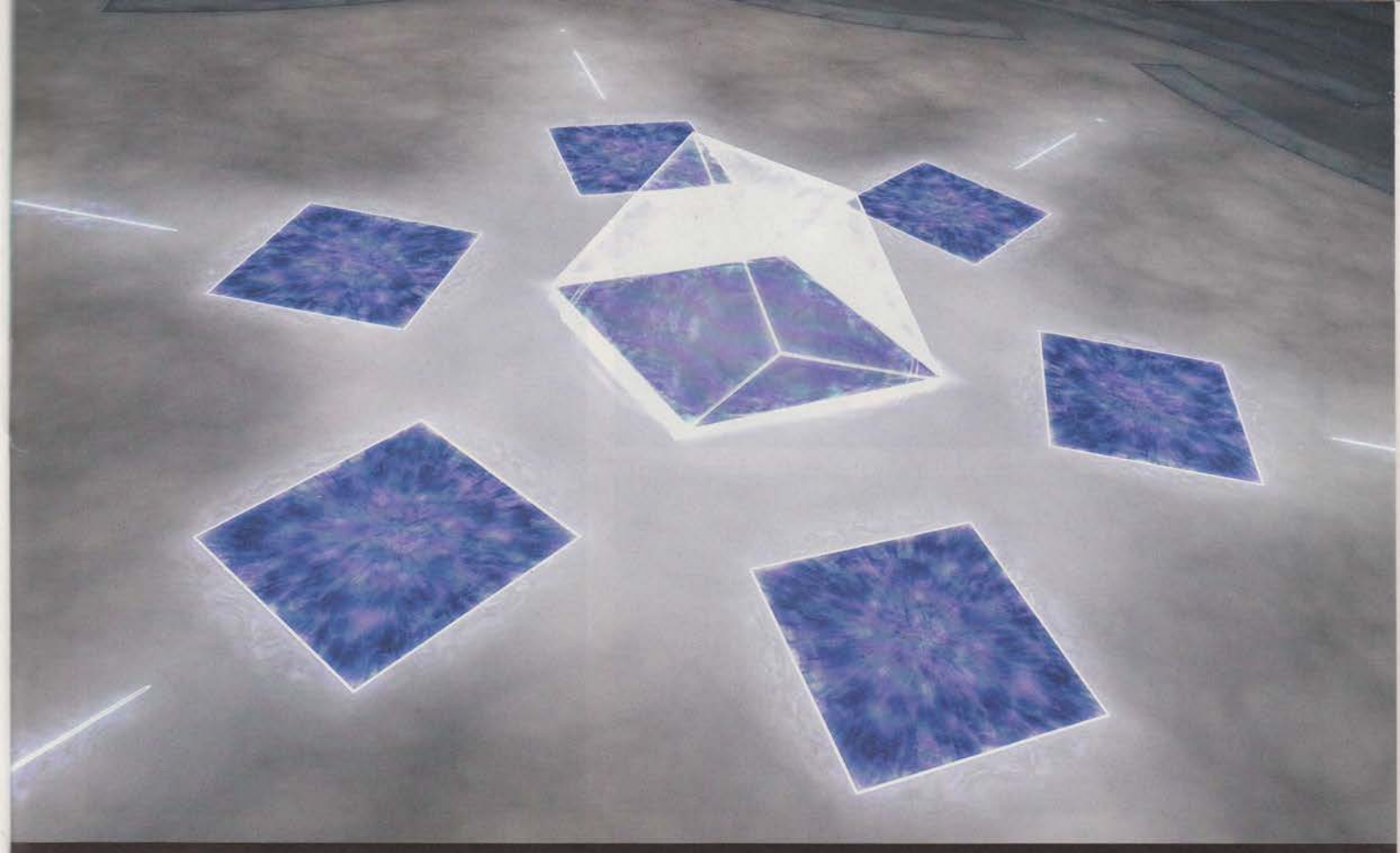

Adding Depth to Geometry through Flatland

Gail Marie Anderson

This classic novel introduces students to core geometry concepts, engages them in spatial visualization, and provides rich, cross-curricular learning opportunities.

Mathematics … reason … imagination … will help reveal the truth.
—Arthur Square, *Flatland: The Movie*

Flatland, by Edwin Abbott, is a fantastic story about a square who lives in a two-dimensional world and who receives a visitor from the third dimension. Written in 1884 by a teacher-theologian who dabbled in mathematics, the novel is full of rich themes, including social class structure, the treatment of people who are different, stereotyping of women, and the existence of higher dimensions that we cannot see. I use the movie version of the book—*Flatland: The Movie* (2007; available at www.flatlandthemovie.com)—in my geometry classes as an introduction to the concepts of points, lines, planes, and space. This is a nice alternative to starting the year off with a load of vocabulary and nonmemorable lessons.

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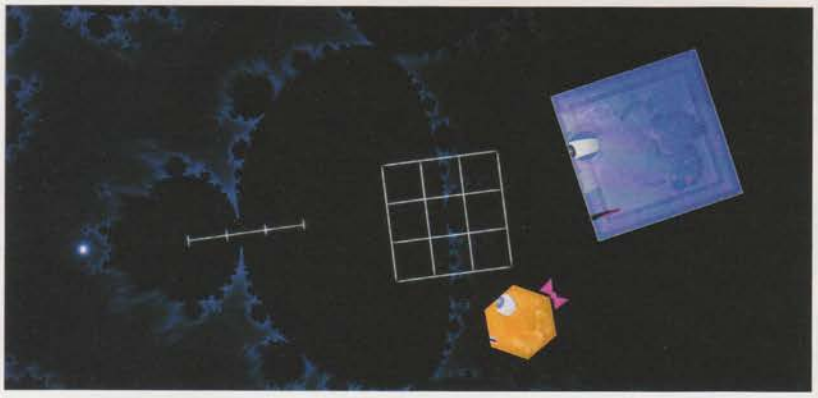

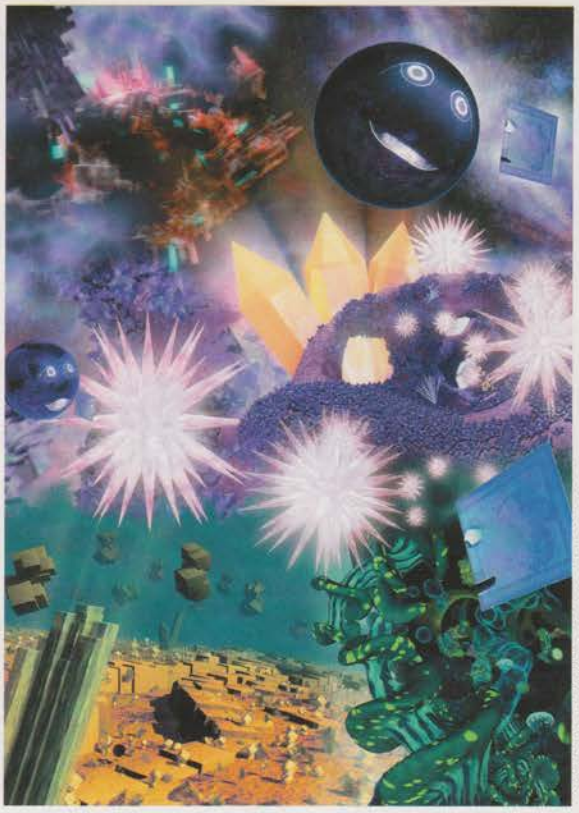

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USING THE MOVIE AND THE BOOK

The movie introduces the nondimensional Pointland, the one-dimensional Lineland, and the two-dimensional Flatland in a way that is easy for students to grasp and remember. Other geometry topics in the film include the intersections of planes with lines, cross sections of three-dimensional objects, movement in one and two dimensions, and an introduction to types of polygons. Two months after the students watched the movie, as we were discussing what a prism is, one student recalled the example given in the movie of taking a polygon and moving it in a direction perpendicular to itself to create a three-dimensional shape. Whenever I tell the class that we are starting an investigation of a new shape, someone asks eagerly, "Is it the hypercube?" Such connections to the movie recur throughout the year.

Having all geometry students read the book (although it is not long) seemed too much to ask in addition to their regularly assigned mathematics homework and the literature that they were already required to read for other classes. I decided to compromise: I showed the movie to all students and required the book only of my honors students. Our classes are mixed-level, providing a nice opportunity for honors students to share what they learned from the novel with the entire class through oral presentations. The presentations included background material on Abbott and the era in which he wrote and explanations of ideas such as how two-dimensional creatures "see" one another. These ideas cannot be developed as well in a movie as in the novel, so the presentations proved valuable for our discussion following the movie.

The movie is only about thirty-five minutes long and so can easily fit into a standard class period if necessary; it also makes a wonderful ninety-minute block period class with the introductions, class discussions, and the included DVD extras on the fourth dimension.

The entire first half of the novel is spent helping the reader understand what life is like in two dimensions. This is a great exercise in visualization. (Of course, a lot of the exercise is lost when watching the movie, which provides all the visualization for viewers.) It does, however, give students an opportunity to think about what *two-dimensional* means. About halfway through the movie, one of my students exclaimed, "Wait! She can't see [the sphere]; it's only a line to her!" Questions during the postmovie discussion included this one: "How can they tell if they're looking at a square or a pentagon … or even a circle?" All three shapes project to a single line in the Flatlanders' vision. One student took this discussion further and observed, "If you think about it, we really only see in two dimensions—we can't see through people, either."

A visit to Flatland by a sphere gives an opportunity for students to learn about planar cross-sections. One extra on the classroom DVD version of the movie includes visualizations of the fourth

dimension, in which the speaker inflates a balloon and then lets it deflate to illustrate a visit by a four-dimensional "supersphere" to our three-dimensional world. The DVD also contains an activity sheet that helps students observe patterns in one-, two-, and three-dimensional objects, patterns that they can then project to define objects in a fourth dimension (a nice challenge for honors students).

This movie also provides a rich source of cross-disciplinary discussion and learning. We discuss how geometry is used to represent the rigid social structure of nineteenth-century England in which Abbott wrote. In Flatland, each type of polygon has an assigned role, ranging from the soldiers who were isosceles triangles, to the office-working squares, to the ruling class of circular "priests." Students were stirred by the treatment of the irregular polygon in the beginning of the film; they easily made the connection to social justice. One student compared the movie to the science fiction novel *The Giver* (Lowry 1993), which she had recently read: "It reminded me of the book *The Giver* because of how [characters] were labeled and what they did for a living. How they are marked and how they live their life." Students experience discomfort when characters judge others by their appearance.

STUDENT REACTIONS

Having this discussion in a geometry classroom gave students a chance to see how mathematics can be useful for describing and understanding the world in which they live. One student commented, "The movie had a lot of parallels in our own world, like how people don't like new ideas." We discussed the fourth dimension (and beyond) and how hard this idea is to accept. There is a difference between believing what we have not seen (which was what the main character, Arthur Square, wanted his people to do) and believing because we have seen and experienced (as Arthur Square did). We can use patterns and projections to begin to comprehend higher, nonintuitive concepts.

My honors students really benefited from both reading the book and watching the movie. They were divided about which version they preferred, but all students agreed on one thing: "The movie really helped me visualize Flatland and the way they see compared to us." Students who have read the book should be prepared for some pretty significant changes made for the movie version. These make for a great discussion topic or writing prompt for students who read the book as well.

The following comments were typical:

- "I want to visit Flatland ☺."
- "Really good movie. It helped me visualize different dimensions."
- "Wasn't at all what I expected; very interesting and different. Better than the other math movies I've watched."
- "I wonder about the 4th dimension and what it would look like.

These comments and reactions indicated that the vast majority of my students enjoyed this learning experience. One former student whose class had not seen the movie stopped by my room to ask why his class had not seen it.

Last, and perhaps most important, the movie serves to prime the imagination. When asked for general comments and reactions to the movie, one student replied, "[It was a] weird way of explaining things, but makes me wonder." Just what we want our students to do!

[Activity sheets for use with the book or movie are available at **www.nctm.org/mt007**. —*Ed.*]

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For downloadable, customizable Word documents for the activity sheets, go to **www.nctm.org/mt007**.

Download one of the free apps for your smartphone. Then scan this tag to access **www.nctm.org/mt007.**